Math 60 10.5 Day 2 Quadratic Functions: Vertex + max/min

Objectives

1) Determine if a quadratic function has a maximum value or a minimum value.

2) Find the maximum or minimum value of a quadratic function.

3) Use the maximum or minimum to solve word problems.

Math 60 10.5 - Day 2

The minimum value of a quadratic function

min

a) is the y-coordinate of the vertex, k

b) is the lowest y-coord (function value)

c) occurs because the parabola opens up. (a>0)

The maximum value of a quadratic function

max


a) is the y-coordinate of the vertex, k

b) is the highest y-coord (function value)

c) occurs because the parabola opens down. (a<0)

A quadratic function has either a max value or a min value, but never both.

Determine if the quadratic function has a maximum value or a minimum value, then find that max/min value and the x-coordinate where it occurs.

(A) $f(x) = 2x^2 + 12x - 3$

$a > 2$ ↑↑ $\boxed{\text{minimum}}$

vertex formula $h = \frac{-b}{2a} = \frac{-12}{2(2)} = \frac{-12}{4} = -3.$

$$k = f(h) = f(-3) = 2(-3)^2 + 12(-3) - 3$$
$$= 2(9) - 36 - 3$$
$$= 18 - 36 - 3$$
$$= -21$$

$\boxed{\text{minimum value is } -21, \text{ it occurs at } x = -3}$

(B) $f(x) = -3x^2 + 6x + 4$

$a = -3$ ↙↘ maximum

$h = \frac{-b}{2a} = \frac{-6}{2(-3)} = 1$ $\quad k = f(1) = -3(1)^2 + 6(1) + 4 = 7$

$\boxed{\text{maximum value is } 7, \text{ it occeurs at } x = 1}$

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Handout:

(1) $f(x) = x^2 - 6x + 3$

$a = 1$ has minimum.

$h = \frac{-b}{2a} = \frac{-(-6)}{2} = \frac{6}{2} = 3$ x-coord of vertex

$f(3) = 3^2 - 6(3) + 3 = -6$

$\boxed{\text{min value } -6}$ at $x = 3$

(2) $A(t) = -t^2 + 4t + 12$

$a = -1$ has maximum

$h = \frac{-b}{2a} = \frac{-4}{2(-1)} = 2$ t-coordinate of vertex

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$$A(2) = -2^2 + 4(2) + 12$$
$$= -4 + 8 + 12$$
$$= 16$$

$\boxed{A(t) \text{ has a maximum value } 16}$ at $t = 2$

③ A contest called Punkin Chunkin is held to see whose device can hurl a pumpkin the furthest.
An air cannon shoots a pumpkin so that its height s in feet at time t seconds after being fired is given by $s(t) = -16t^2 + 240t + 10$.

a) Determine the time at which the pumpkin is at its maximum height.

vertex formula
$$h = -\frac{b}{2a} = -\frac{240}{2(-16)} = 7.5$$ ← t-coordinate of the vertex

$\boxed{\text{At time } t = 7.5 \text{ seconds, pumpkin reaches max height.}}$

b) Determine the maximum height of the pumpkin.

vertex formula
$$k = s(h) = s(7.5)$$
$$= -16(7.5)^2 + 240(7.5) + 10$$
$$= \boxed{910 \text{ feet}}$$ max height.

c) After how long will the pumpkin strike the ground? (Round to nearest hundredth of a second.)

"strike ground" means height $= 0$
set $s(t) = 0$.

Math 60 10.5 - 2nd

$$0 = -16t^2 + 240t + 10$$

This is a quadratic equation - chapter 9!
We can solve by
- factoring
- CTS & square root property
- quadratic formula

You can solve the equation as given, but it's probably easier if we make the numbers smaller.

$$\frac{-16t^2}{-2} + \frac{240t}{-2} + \frac{10}{-2} = \frac{0}{-2}$$

$$8t^2 - 120t - 5 = 0$$

$$D = b^2 - 4ac$$
$$= (-120)^2 - 4(8)(-5)$$
$$= 14560$$
$$\sqrt{14560} \approx 120.66\ldots$$

It's not a perfect square. So the equation cannot be solved by factoring.

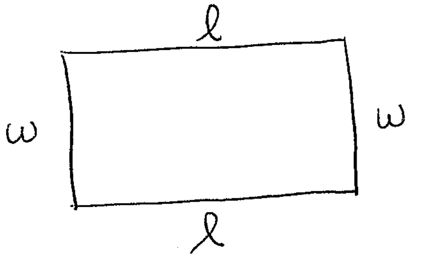
Quadratic formula

$$t = \frac{-(-120) \pm \sqrt{14560}}{2(8)}$$

$\boxed{t \approx 15.04 \text{ sec}}$ or ~~$t \approx -.04$ sec extraneous.~~

Math 60 10.5 - 2nd

#5) A farmer has 3000 ft of fence with which to enclose a rectangular field. What is the maximum area that can be enclosed by the fence? What are the dimensions of the field with maximum area?



Perimeter of field = amount of fencing available

$$2l + 2w = 3000$$

In the question: "maximum area" tells us we want a quadratic function giving area.

Area of a rectangle $A = l \cdot w$

too many different variables!

Use $2l + 2w = 3000$ to isolate one variable.

I choose l:

$$\frac{2l}{2} = \frac{3000}{2} - \frac{2w}{2}$$

$$l = 1500 - w.$$

Replace l in $A = l \cdot w$ by $(1500 - w)$. (Substitute!)

$$A = (1500 - w) \cdot w$$

Distribute $A = 1500w - w^2.$

Write in familar notation: $A \rightarrow f(x)$ — y coordinate

$w \rightarrow x$

$$f(x) = 1500x - x^2$$

Rearrange $f(x) = -x^2 + 1500x.$ $a = -1$ minimum

vertex formula $h = \frac{-b}{2a} = \frac{-1500}{2(-1)} = 750 \leftarrow$ x coordinate of vertex

$$k = f(750) = -(750)^2 + 1500(750) = 562500$$

maximum area $A \rightarrow f(x) \rightarrow y = \boxed{562500 \text{ sq ft max area}}$

occurs when $\boxed{x = w = 750 \text{ ft.}}$ and $\boxed{l = 1500 - 750 = 750 \text{ ft}}$

Math 60 10.5 - 2nd

(5) The marketing department says revenue from selling a computer at price p dollars is $R(p) = -\frac{1}{4}p^2 + 400p$

a) For what price will revenue be maximized?

b) What is the maximum revenue?

$$R(p) = -\frac{1}{4}p^2 + 400p$$

change to familar notation $\quad p \to x$, $R \to f$

$$f(x) = -\frac{1}{4}x^2 + 400x$$

vertex formula $\quad h = \frac{-b}{2a} = \frac{-400}{2(-1/4)} = 800 \to x \to p$

$\boxed{\text{price } \$800}$

$$k = f(h) = f(800) = -\frac{1}{4}(800)^2 + 400(800)$$

$= \boxed{\$160\,000 \text{ revenue}} \leftarrow f \leftarrow R \leftarrow$ revenue

Math 60 10.5 -2nd

(6) A tour company sells tickets for \$90 each for the first 30 people. For larger groups, the price per ticket is reduced by \$0.50 for all members of the group. How many tickets will maximize revenue? What is the maximum revenue?

"Revenue" means "money received". It does not include costs or any other deductions.

Revenue = (# people) times (price per ticket).

Since revenue goes up after 30 tickets are sold, the maximum revenue occurs for a group size more than 30.

Let x = # people over 30 in the group. ← ex: group is 32 people, then $x = 2$

Then # people in group = $(30 + x)$.

Ticket price = \$90 − (\$0.50 for each person more than 30 in group)

= \$90 − \$0.50x

Revenue R = (# people) × (price per person)

$$R = (30 + x)(90 - .5x)$$

Familiar name: $R \to f(x)$.

$$f(x) = (30 + x)(90 - .5x)$$

FOIL $f(x) = 2700 - 15x + 90x - .5x^2$

rearrange $f(x) = -0.5x^2 + 75x + 2700$

vertex formula $h = \frac{-b}{2a} = \frac{-75}{2(-0.5)} = 75$ ← x coord of vertex

people = $30 + x \Rightarrow 30 + 75 = \boxed{105 \text{ tickets sold}}$

Maximum revenue → max f → max y → y coord of vertex

$$f(75) = -0.5(75)^2 + 75(75) + 2700$$

$$= \boxed{\$5512.5 \text{ maximum revenue}}$$

Math 60: 10.5 Word Problems

Identify if the function has a maximum or minimum value, then find that value.

1) $f(x) = x^2 - 6x + 3$

2) $A(t) = -t^2 + 4t + 12$

3) A contest called Punkin Chunkin is held to see whose device can hurl a pumpkin the furthest. An air cannon shoots the pumpkin so that its height s in feet at time t seconds after being fired is given by $s(t) = -16t^2 + 240t + 10$

   a. Determine the time at which the pumpkin is at its maximum height.
   b. Determine the maximum height of the pumpkin.
   c. After how long will the pumpkin strike the ground? (Round to the nearest hundredth of a second.)

4) A farmer has 3000 feet of fencing with which to enclose a rectangular field.

   a. What is the maximum area that can be enclosed by the fence?
   b. What are the dimensions of the field with maximum area?

5) The marketing department says revenue from selling a computer at price p dollars is $R(p) = -\frac{1}{4}p^2 + 400p$.

   a. For what price will revenue be maximized?
   b. What is the maximum revenue?